THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Advanced Optimization, Spring 2025
Department of Statistics and Operations Research

Instructor:

- Ali Mohammad Nezhad
- Email: alimn@unc.edu
- Homepage: `https://alimn.stor.unc.edu/index.html`
- Office: Hanes 330

- Office hours: Tuesday and Thursday 04:00 - 05:00 PM

Lecture:

- STOR 614-001 (1639): Time: Tuesday and Thursday 02:00 - 03:15 PM Location: HN 130

Course homepage: `https://alimn.stor.unc.edu/STOR-614.html`

Canvas: `https://edtech.unc.edu/service/canvas/`

Teaching Assistants:

- Yanhui Ren (yar@unc.edu)

Course Description

Lectures

This is a first course on the theory of nonlinear optimization, with emphasis on semi-definite and polynomial optimization. The main focus will be on the theory with proofs (duality, optimality, attainment, sensitivity analysis, complexity), algorithmic development (e.g., interior point methods), and their complexity and proof of convergence (e.g., using error bounds and Łojasiewicz inequality). We also highlight applications to applied mathematics, computer science, statistics, data science, and other fields. This course is designed for graduate students in Statistics and Analytics, Mathematics, and Computer Sciences. This is a self-contained course, but I assume that the students are familiar with basic linear algebra, multivariate calculus, and basic mathematical analysis (e.g., proof techniques). I will always provide the necessary background and from algebra, analysis, and topology whenever needed.

I teach by writing on the blackboard. The tentative course schedule can be found on the course webpage.

- I will upload the lecture notes to Canvas.

- If we are not able to meet in person (e.g., due to an unprecedented situation or a decision made by the university), then we may continue with the lectures as scheduled over Zoom. Details will be announced on Canvas.

References: The course materials are (almost always) selected from the following references (none of which is required for this course):

- A. V. Fiacco and G. P. McCormick. Nonlinear Programming: Sequential Unconstrained Minimization Techniques. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1990.
- **Recommended**: E. de Klerk. Aspects of Semidefinite Programming. Interior Point Algorithms and Selected Applications, volume 65 of Series Applied Optimization. Springer, New York, NY, USA, 2006.
- **Recommended**: M. S. Bazaraa, H. D. Sherali, C. M. Shetty. Nonlinear Programming. Theory and Algorithms, third edition. Wiley-Interscience, NH, USA, 2006.
- **Recommended**: A. Ben-Tal and A. Nemirovski. Lectures on Modern Convex Optimization. Analysis, Algorithms, and Engineering Applications. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2001.
- **Recommended**: S. J. Wright, Primal-dual interior-point methods, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1997.
- C. Roos, T. Terlaky, J-P Vial. Interior Point Methods for Linear Optimization, second edition, Springer, New York, NY, USA, 2005.
- M. Laurent. Sums of Squares, Moment Matrices and Optimization Over Polynomials. In Emerging Applications of Algebraic Geometry, editors M. Putinar and S. Sullivant. Springer, New York, NY, USA, 2009.
- J. B. Lasserre, An introduction to polynomial and semi-algebraic optimization, Cambridge Texts in Applied Mathematics, Cambridge University Press, Cambridge, 2015.
- S. Basu, R. Pollack, and M.-F. Roy. Algorithms in Real Algebraic Geometry. Springer, New York, NY, USA, 2006.

The following topics will be covered (not necessarily in the same order):

- **Nonlinear optimization theory**: formulation and applications, critical conditions, KKT points, Lagrangian duality and saddle points, Lagrange multipliers, critical points on manifolds, constraint qualifications, attainment of optimal values, implicit function theorem and sensitivity analysis, connection to Morse theory, error bounds and Łojasiewicz inequality
- **Theory of conic optimization (with emphasis on semi-definite optimization)**: duality theory and optimality, degeneracy, central path, optimal partition, sensitivity analysis
- **Algorithms for conic optimization**: affine scaling methods, primal-dual path-following interior point methods
- **Applications**: applications of conic optimization to combinatorial optimization, polynomial optimization, and sums of squares
- **Advanced topics (if time permits)**: introduction to polynomial optimization and real algebraic geometry, computational complexity of polynomial optimization, representation of positive polynomials, symbolic computation

Learning Objectives

By successfully completing this course, you will

- learn the duality, complexity, degeneracy, and few other complications in nonlinear optimization (versus linear optimization)
- learn the basics of sensitivity analysis in nonlinear and semi-definite optimization
- understand how and why nonlinear optimization algorithms converge and with what rate
- understand the notion of computational complexity in polynomial and semi-definite optimization through the lens of real algebraic geometry

Overall, upon a successful completion of this course, you will enhance your algebraic understanding of stability and computational complexity in optimization.

Assignments

You will be given handwritten homework assignments, almost every week (depending on the instructor). The problems will be posted on Gradescope. You need to write down the solution(s) to the specified questions on a paper, create a scanned copy of your solution in PDF format (or you can use MS Word/Latex or any similar interface on your laptop/iPad/Tablet) and then upload the PDF file to Gradescope. An introductory document for Gradescope can be found on the course webpage.

- The deadlines will be always announced to you via Canvas.
- Your two lowest scores from the homework assignments will be dropped.

Exams

There will be one (1) 75-min in-class midterm examination and a 3-hour in-class final exam as follows:

Midterm Exam:	Date:	Tuesday 03/04/2025	Time:	02:00 - 03:15 PM	Location: HN 130
Final Exam:	Date:	Thursday 05/01/2025	Time:	12:00 - 03:00 PM	Location: HN 130

- All exams will be closed book and closed notes.

Grading Policy

Your final grade is calculated based on the following formula:

Homework	20%
Midterm exam	40%
Final exam	40%

Your total score = Homework $\times$ 0.2 + max{Midterm, Final} $\times$ 0.60 + min{Midterm, Final} $\times$ 0.2.

How to succeed in this course

You are expected to attend and be actively involved in all classes and take the homework assignments very seriously and complete them by the deadline.

- You are strongly encouraged to attend the class on a regular basis and be always on time. You are welcome to raise your hand and ask questions if anything comes to your mind. Please do not talk to the students around you as this could cause distraction.

Course Policy

You are allowed to work on the homework problems together and feel free to use the references that I mentioned or any other textbook that can be useful. However, you are not allowed to use online resources. Further, you must write down your solutions in your own words all by yourself. In case of collaboration, you should explicitly cite the references that you used or spell out the extent of help and your classmate(s) that you worked with on the problems. No points will be taken off for such disclosures. However, you will get zero on an assignment, if your solution is recognized to be identical to that of another student, or if you provide a solution which is not yours. Please also see the section on Honor Code Statement.

- Please note that a frequent unauthorized use of help will be more a learning issue rather than a cheating issue. If you always get the answers from other sources it will affect your learning.

Deadline: A homework will be due in a week or so at 11:59 PM.

- Late submissions are NOT accepted. If you are not able to meet a deadline (due to illness or a university-approved excuse) and you need an extension, please contact me at your earliest convenience before the deadline. I then will decide on a case by case basis.

Use of Electronic Devices You are more than welcome to use your electronic devices during the lectures, but only for this class purposes (e.g., note taking). Please note that an inappropriate use of your electronic devices will cause distraction for the students around you.

Make-up Exams: If you are not able to attend an exam due to an illness, in the event of an emergency or other reasons, 1) you need to contact me at your earliest convenience before the exam, and 2) you need to provide a university-approved letter (a letter from your doctor or primary care provider is also acceptable). In that case, I will schedule a make-up exam for you. Otherwise, your absence will not be acceptable, and your grade on that exam will be zero.

Regrade Request You have two weeks from the release date of the grades to submit a regrade request on Gradescope. No further requests will be accepted after this deadline.

Announcements: Please stay tuned for announcements in Canvas. All course related information, including the deadline for all assignments, will be announced in Canvas.

Syllabus Changes

I reserve the right to make changes to the syllabus, including due dates and test dates. These changes will be announced as early as possible.

Honor Code Statement

Students are bound by the Honor Code in taking exams and in written work, that includes issues such as plagiarism, falsification, unauthorized assistance, cheating, and other acts of academic dishonesty (please see also the course policy). The Honor Code of the University is in effect at all times, and the submission of work signifies understanding and acceptance of those requirements. Posting homework assignments, exam problems, or any part of them on the internet, or copying solutions from internet resources is a violation of the Honor Code. Please consult with me if you have any questions about the Honor Code, or you can read more about the Honor Code at `studentconduct.unc.edu`.

Accessibility Resources and Services

I strive to make learning experiences accessible to all students. If you anticipate or experience physical or academic barriers based on disability or you need any accommodations, you are welcome to let me know so that we can discuss options. Please also be advised that the University of North Carolina at Chapel Hill facilitates the implementation of reasonable accommodations, including resources and services, for students with disabilities, chronic medical conditions, a temporary disability or pregnancy complications resulting in barriers to fully accessing University courses, programs and activities. Accommodations are determined through the Office of Accessibility Resources and Service (ARS) for individuals with documented qualifying disabilities in accordance with applicable state and federal laws. See the ARS website `https://ars.unc.edu`. For contact information email ars@unc.edu.

Tests for ARS students

If you have been certified by ARS as eligible for accommodations, then you should contact me to discuss your accommodations as soon as possible. I will work with you to ensure that accommodations are provided as appropriate. For quizzes, you get extended time at the end of the class (unless the accommodations say that you need a separate room too). You will take the exams at ARS. An exam at ARS must be scheduled at the same time as the regular exam.

Equal Opportunity and Compliance Accommodations

Equal Opportunity and Compliance Accommodations Team (Accommodations – UNC Equal Opportunity and Compliance) receives requests for accommodations for disability, pregnancy and related conditions, and sincerely held religious beliefs and practices through the University's Policy on Accommodations. EOC Accommodations team determines eligibility and reasonable accommodations consistent with state and federal laws.

Counseling and Psychological Services

UNC-Chapel Hill is strongly committed to addressing the mental health needs of a diverse student body. The Heels Care Network website is a place to access the many mental health resources at Carolina. CAPS is the primary mental health provider for students, offering timely access to consultation and connection to clinically appropriate services. Go to their website or visit their facilities on the third floor of the Campus Health building for an initial evaluation to learn more. Students can also call CAPS 24/7 at 919-966-3658 for immediate assistance.

Title IX Resources

Any student who is impacted by discrimination, harassment, interpersonal (relationship) violence, sexual violence, sexual exploitation, or stalking is encouraged to seek resources on campus or in the community. Reports can be made online to the EOC at `https://eoc.unc.edu/report-an-incident/` or by contacting the University's Title IX Coordinator (Elizabeth Hall, titleixcoordinator@unc.edu) or the Report and Response Coordinators in the Equal Opportunity and Compliance Office (reportandresponse@unc.edu). Confidential resources include Counseling and Psychological Services and the Gender Violence Services Coordinators (gvsc@unc.edu). Additional resources are available at `safe.unc.edu`.

Attendance Policy

As stated in the University's Class Attendance Policy, no right or privilege exists that permits a student to be absent from any class meetings, except for these University Approved Absences:

1. Authorized University activities: University Approved Absence Office (UAAO) website provides information and FAQs for students and FAQs for faculty related to University Approved Absences.
2. Disability/religious observance/pregnancy, as required by law and approved by Accessibility Resources and Service and/or the Equal Opportunity and Compliance Office (EOC).
3. Significant health condition and/or personal/family emergency as approved by the Office of the Dean of Students, Gender Violence Service Coordinators, and/or the Equal Opportunity and Compliance Office (EOC).

I work with students to meet attendance needs that do not fall within University approved absences. For situations when an absence is not University approved (e.g., a job interview, illness/flu, or club activity), I will work directly with you to determine the best approach to missed classes and make-up assessment and assignments.

Acceptable Use Policy

By attending the University of North Carolina at Chapel Hill, you agree to abide by the University of North Carolina at Chapel Hill policies related to the acceptable use of IT systems and services. The Acceptable Use Policy (AUP) sets the expectation that you will use the University's technology resources responsibly, consistent with the University's mission. In the context of a class, it's quite likely you will participate in online activities that could include personal information about you or your peers, and the AUP addresses your obligations to protect the privacy of class participants. In addition, the AUP addresses matters of others' intellectual property, including copyright. These are only a couple of typical examples, so you should consult the full Information Technology Acceptable Use Policy, which covers topics related to using digital resources, such as privacy, confidentiality, and intellectual property.

Additionally, consult the Safe Computing at UNC website for information about data security policies, updates, and tips on keeping your identity, information, and devices safe.

Diversity Statement

I value the perspectives of individuals from all backgrounds reflecting the diversity of our students. I broadly define diversity to include race, gender identity, national origin, ethnicity, religion, social class, age, sexual orientation, political background, and physical and learning ability. I strive to make this classroom an inclusive space for all students. Please let me know if there is anything I can do to improve. I appreciate any suggestions.

Policy on Non-Discrimination

The University is committed to providing an inclusive and welcoming environment for all members of our community and to ensuring that educational and employment decisions are based on individuals' abilities and qualifications. Consistent with this principle and applicable laws, the University's Policy Statement on Non-Discrimination offers access to its educational programs and activities as well as employment terms and conditions without respect to race, color, gender, national origin, age, religion, genetic information, disability, veteran's status, sexual orientation, gender identity or gender expression. Such a policy ensures that only relevant factors are considered, and that equitable and consistent standards of conduct and performance are applied.

If you are experiencing harassment or discrimination, you can seek assistance and file a report through the Report and Response Coordinators (email reportandresponse@unc.edu or see additional contact info at `safe.unc.edu`) or the Equal Opportunity and Compliance Office at `https://eoc.unc.edu/report-an-incident/`.

Policy on Use of Artificial Intelligence

Use of generative AI tools of any kind is not permitted in this course. Any use of these tools will be considered an instance of academic dishonesty and will be referred to Student Conduct.